

CHAPTER 4. PROBABILITIES AND COUNTING RULES

SAMPLE TEST

1) A box contains 12 red balls, 8 yellow balls and 4 purple balls. If one ball is selected at random, find the probability:

- a) to select a red marble
- b) to select a yellow marble
- c) to select not a purple marble

2) A fair six-sided die is rolled, find the probability:

- a) to roll a 5
- b) to roll an even number
- c) to roll a number greater than 6

3) Two fair six-sided dice are rolled one time, find the probability:

- a) to roll a sum of 10
- b) to roll a sum of 7
- c) to roll a sum of 4
- d) to roll a sum of 12
- e) to roll a sum of 14
- f) to roll a sum greater than 9
- g) to roll a sum less than or equal to 5

4) Each of the letters in the word **SAMSUNG** are on separate cards face down on the table. If you pick a card at random, what is the probability that its letter will be:

- a) N
- b) S
- c) A or S

5) Karen is playing cards with her friend when she draws a card from a pack of 20 cards numbered from 1 to 20. What is the probability of drawing:

- a) number divisible by 5
- b) number divisible by 3
- c) even number

6) Suppose you select a card from a standard deck of cards, find the probability:

- a) to select a **red** card
- b) to select a K
- c) to select a red **10**
- b) to select a black **K**
- e) to select a 6

7) Suppose you select a card from a standard deck of cards, find the probability

- a) to select a Q or a J
- b) to select a face card (J, Q, K)
- c) to select a red **5** or a black **9**

8) Suppose you select a card from a standard deck of cards, find the probability:

- a) to select a **red** card or a Q
- b) to select a **black** card or a K

9) Two cards are pulled in order, *without replacement* from this deck of 52 playing cards.
What is the probability that the cards will be an 8 of hearts and then an Ace?

10) Two cards are pulled in order, *without replacement* from this deck of 52 playing cards.
What is the probability that the cards will be a **red** K and then a Q?

11) Two cards are pulled in order, *without replacement* from this deck of 52 playing cards.
What is the probability that the cards will be a 10 and then a J?

12) Two cards are pulled in order, *without replacement* from this deck of 52 playing cards.
What is the probability that the two cards will be **red J**?

13) Three cards are pulled in order, *with replacement* from this deck of 52 playing cards.
What is the probability that the three cards will be **red**?

14) Three cards are pulled in order, *with replacement* from this deck of 52 playing cards.
What is the probability that the cards will be K, then a Q, and then J?

15) A fair six-sided die is rolled, find the probability:

- a) to roll a 6 or a number greater than 4
- b) to roll a 4 or an even number

16) A die is rolled 3 times. What is the probability:

- a) to roll no 5
- b) to roll 5 one time
- c) to roll 5 two times
- d) to roll 5 three times

17) In a sample of 50 people the following frequency distribution of blood types was found, If one person is selected at random find the probability:

- a) the person has A blood type
- b) the person has B blood type
- c) the person has blood type AB or O

Type	Frequency
A	22
B	5
AB	2
O	21

18) Which of the following cannot be a probability

- 0.78	5/6	65%	9/8	0.912	135%	- 0.45	1.02	0.65%	0.0006
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19) If a number is randomly selected chosen from the following list of numbers. What is the probability that the number is multiple of 5?

[14, 40, 32, 56, 75, 49, 22, 50, 30, 6]

20) What is the probability of choosing the letter **i** from the word **probability**?

21) What is the probability of landing on an odd number after spinning a spinner with 9 equal sectors numbered 1 through 9?

22) The probability that it is Tuesday and that a student is absent is 0.05. Since there are 5 school days in a week, the probability that it is Tuesday is 0.2. What is the probability that a student is absent given that today is Tuesday?

23) A nationwide survey showed that 62% of all children in the United States dislike eating vegetables. If 4 children are chosen at random, what is the probability that all 4 dislike eating vegetables?

24) A number from 1 to 12 is chosen at random. What is the probability of choosing a 5 or an even number?

25) A jar contains black and white marbles. Two marbles are chosen without replacement. The probability of selecting a black marble and then a white marble is 0.34, and the probability of selecting a black marble on the first draw is 0.47. What is the probability of selecting a white marble on the second draw, given that the first marble drawn was black?

26) In New York State, 52% of all teenagers own a bicycle and 29% of all teenagers

27) What is the probability of choosing the letter **t** from the word **Statistics**?

28) A jar contains 6 red cards, 4 yellow cards, and 5 blue cards. If one card is selected at random, what is the probability

- a) to select a red card
- b) to select a blue card
- c) to select a yellow or a red card

29) A jar contains 6 red cards, 4 yellow cards, and 5 blue cards. If two cards are selected at random, without replace the first, what is the probability

- a) to select a 2 red cards
- b) to select a 2 yellow cards
- c) to select a yellow a then a red card

30) The following table presents the probability of the numbers of defects in a randomly printed – circuit board.

Number of defects	0	1	2	3	4
Percent of defects	25%	22%	20%	18%	15%

- Find the probability that the numbers of defects will be at least 2.
- Find the probability that the numbers of defects will be fewer than 3.
- Find the probability that the numbers of defects will be at most 3.
- Find the probability that the numbers of defects will be more than 3.

31) Evaluate the expression:

- $10P5$
- $9C4$
- $6!$

32) There are 11 members on a board of directors. If they must form a subcommittee of 5 members, how many different subcommittees are possible?

33) How many ways can an IRS auditor select 6 of 12 tax returns for an audit?

34) There are 6 members on a board of directors. If they must elect a chairperson, a secretary, and a treasurer, how many different slates of candidates are possible?

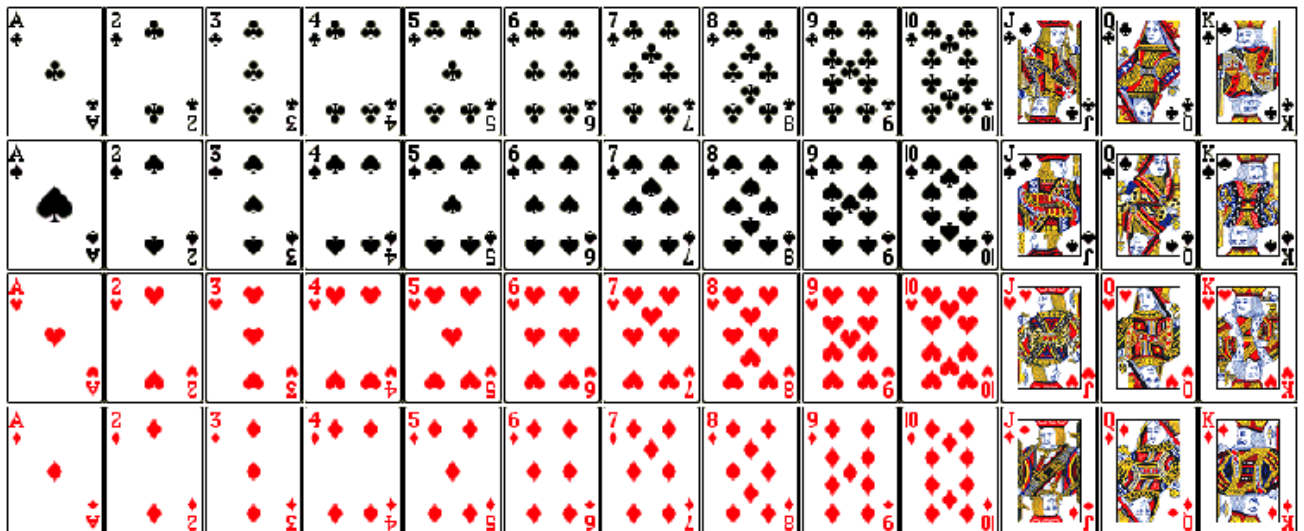
35) A restaurant offers a choice of 4 salads, 8 main courses, and 4 desserts. How many possible 3 - course meals are there?

Answer for all the exercises in the next pages

Standard deck of cards with **52** cards

26 black cards -----13 (♠) spades and 13 (♣) clubs

26 red cards ----- 13 (♥) hearts and +13 (♦) diamonds



Sample space when we roll two dice

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

ANSWERS

1)

a) $12/24 = 1/2$

b) $8/24 = 1/3$

c) $12/24 + 8/24 = 20/24 = 5/6$

2)

a) $1/6$

b) $3/6 = 1/2$

c) $0/6 = 0$

3)

a) $3/36 = 1/12$

b) $6/36 = 1/6$

c) $3/36 = 1/12$

d) $1/36$

e) $0/36 = 0$

f) $3/36 + 2/36 + 1/36 = 6/36 = 1/6$

g) $1/36 + 2/36 + 3/36 + 4/36 = 10/36 = 5/18$

4)

a) $1/7$

b) $2/7$

c) $3/7$

5)

a) $4/20 = 1/5$

b) $6/20 = 3/10$

c) $10/20 = 1/2$

6)

a) $26/52 = 1/2$

b) $4/52 = 1/13$

c) $2/52 = 1/26$

d) $2/52 = 1/26$

e) $4/52 = 1/13$

7)

a) $4/52 + 4/52 = 8/52 = 2/13$

b) $4/52 + 4/52 + 4/52 = 12/52 = 3/13$

c) $2/52 + 2/52 = 4/52 = 1/13$

8)

a) $26/52 + 4/52 - 2/52 = 28/52 = 7/13$

b) $26/52 + 4/52 - 2/52 = 28/52 = 7/13$

9) $(1/52) * (4/51) = 1/663$

10) $(2/52) * (4/51) = 2/663$

11) $(4/52) * (4/51) = 4/663$

12) $(2/52) * (1/51) = 1/1326$

13) $(26/52) * (26/52) * (26/52) = (1/2) * (1/2) * (1/2) = 1/8$

14) $(4/52) * (4/52) * (4/52) = 1/2197$

15)

a) $1/6 + 2/6 - 1/6 = 2/6 = 1/3$

Note: We have to subtract 1/6 because we are counting the 6 twice

b) $1/6 + 3/6 - 1/6 = 3/6 = 1/2$

Note: We have to subtract 1/6 because we are counting the 4 twice

16)

a) $\left(\frac{5}{6}\right) * \left(\frac{5}{6}\right) * \left(\frac{5}{6}\right) = 125/216$

b) $\left(\frac{1}{6}\right) * \left(\frac{5}{6}\right) * \left(\frac{5}{6}\right) + \left(\frac{5}{6}\right) * \left(\frac{1}{6}\right) * \left(\frac{5}{6}\right) + \left(\frac{5}{6}\right) * \left(\frac{5}{6}\right) * \left(\frac{1}{6}\right) = 75/216$

c) $\left(\frac{1}{6}\right) * \left(\frac{1}{6}\right) * \left(\frac{5}{6}\right) + \left(\frac{1}{6}\right) * \left(\frac{5}{6}\right) * \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right) * \left(\frac{1}{6}\right) * \left(\frac{1}{6}\right) = 15/216$

c) $\left(\frac{1}{6}\right) * \left(\frac{1}{6}\right) * \left(\frac{1}{6}\right) = 1/216$

Note: The sum of probabilities on sections a, b, c, and d must be one

17)

a) $22/50 = 11/25$

b) $5/50 = 1/10$

c) $2/50 + 21/50 = 23/50$

18) $-0.78, 9/8, 135\%, -0.45, 1.02$

Note: $0 \leq P(\text{of any event}) \leq 1$

19) $4/10 = 2/5$

Note: there are 4 numbers multiple of 5 in the list of 10 numbers)

20) $2/11$

Note: there are 2 l in the word probability

21) $5/9$

Note: there are 5 odd numbers in the spinner (1, 3, 5, 7, and 9)

22) $0.05/0.2 = 0.25$

Note: $P(\text{Absent/ Tuesday}) = P(\text{Absent and Tuesday}/P(\text{Tuesday}))$

23) $0.62 \times 0.62 \times 0.62 \times 0.62 = 0.148$

24) $1/12 + 6/12 = 7/12$

25) $0.34/0.47 = 0.72$

26) $0.29/0.52 = 0.56$

27) $3/10$

28)

a) $6/15 = 2/5$

b) $5/15 = 1/3$

c) $4/15 + 6/15 = 10/15 = 2/3$

29)

a) $(6/15)(5/14) = 1/7$ *Note: Initially there were 6 red cards and 15 cards in all, but after the first red was selected, there were only 5 red cards and 14 cards in all.*

b) $(4/15)(3/14) = 2/35$ *Note: Initially there were 4 yellow cards and 15 cards in all, but after the first yellow was selected, there were only 3 red cards and 14 cards in all.*

c) $(4/15)(6/14) = 4/35$ *Note: Initially there were 4 yellow cards and 15 cards in all, but after the first yellow was selected, there were the same 6 red cards but only 14 cards in all.*

30)

Number of defects	0	1	2	3	4
Percent of defects	25%	22%	20%	18%	15%

a) Find the probability that the numbers of defects will be at least 2.

$$P(\text{at least } 2) = P(2) + P(3) + P(4) = 20\% + 18\% + 15\% = 53\%$$

Include the number 2 when is "at least"

b) Find the probability that the numbers of defects will be fewer than 2.

$$P(\text{fewer than } 2) = P(0) + P(1) = 25\% + 22\% = 47\%$$

It does not include the number 2 when is "fewer than"

c) Find the probability that the numbers of defects will be at most 3.

$$P(\text{at most } 3) = P(0) + P(1) + P(2) + P(3) = 25\% + 22\% + 20\% + 18\% = 85\%$$

Include the number 3 when is "at most"

d) Find the probability that the numbers of defects will be more than 3.

$$P(\text{more than } 3) = P(4) = 15\%$$

It does not include the number 3 when is "more than"

31) Evaluate the expression

$$\text{a) } {}_{10}P_5 = \frac{10!}{(10-5)!} = 30,240$$

$$\text{b) } {}_9C_4 = \frac{9!}{(9-4)!4!} = 126$$

$$\text{c) } 6! = 720$$

$$\text{32) } {}_{11}C_5 = \frac{11!}{(11-5)!5!} = 462 \quad \text{The order does not matter.}$$

$$\text{33) } {}_{12}C_6 = \frac{12!}{(12-6)!6!} = 924 \quad \text{The order does not matter.}$$

$$\text{34) } {}_6P_3 = \frac{6!}{(6-3)!} = 120 \quad \text{The order matters.}$$

$$\text{35) } 8 * 4 * 4 = 128 \quad \text{There will be 128 different three - course meals.}$$